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Energy Losses (Gains) of Massive Coloured Particles in Stochastic Colour Medium

Andrei Leonidov[1]

Physics Department
University of Bielefeld 33615 Bielefeld,Germany
and
Theoretical Physics Department
P.N.Lebedev Physics Institute, 117924 Leninsky pr.53, Moscow, Russia

Abstract

The propagation of massive coloured particles in stochastic background chromoelectric field is studied using the semiclassical equations of motion. Depending on the nature of the stochastic background we obtain the formulae for the energy losses of heavy coloured projectile in nonperturbative hadronic medium and for the energy gains in the stochastic field present, e.g., in the turbulent plasma. The result appears to be significantly dependent on the form of the correlation function of stochastic external field.

[1] Alexander von Humboldt Fellow

1. It has become a part of the standard QCD wisdom that the physics of heavy quarks provides unique possibilities for studying both perturbative and non-perturbative phenomena. From the point of view of perturbation theory, due to the asymptotic freedom the processes involving heavy quarks tend to evolve at small distances [1]. At the same time the heavy quarkonia are a unique testing device for studying the nonperturbative effects. One of the best examples is the appearence of the vacuum condensates in the description of heavy quarkonia physics in the framework of sum rules method [2].

Of a particular interest to high energy physics are the unique abilities of heavy quarkonia to probe the properties of a complicated medium arising in the high energy nucleus-nucleus and hadron-nucleus collisions. In particular charmonium, when propagating through the quark-gluon plasma, effectively measures its temperature giving a possibility of its determination through the J/Ψ to J/Ψ' ratio [3]. Another sensitive point, which was a subject of recent discussions, is a drastic difference between the interaction of heavy quarkonium with the medium, when the heavy quarkonium is in the singlet and octet colour states respectively (see e.g.[4]). In particular, such effects can be very important for the description of the hard region of spectra in the longitudinal momenta of charmonium, produced in hadron-nucleus collisions [5]. Roughly speaking, the small dipole interaction of the singlet projectile with the coloured medium is supposed to be responsible for colour transparency, and that of an octet one for colour opacity. Let us note, that the production of an initial octet then emitting a gluon and becoming a singlet constitutes a part of the perturbative contribution to the production of the heavy vector mesons [6].

Another situation of considerable interest to the high energy physics is a propagation of a heavy quark through the hadronic (nuclear) medium. As the process of creation of heavy quarks can often be reliably calculated in perturbative QCD, one hopes to get an information on the medium through which the heavy quark (heavy quark jet) propagates.

One of the models for the interior of the hadronic medium is that of the soft stochastic colour fields giving rise to the phenomenologically introduced condensates. Different variants of this model were successfully applied both for the explanation of the origin of confinement [7] and for more practical calculations such as those of the meson spectra (see e.g. [8,9]) and of the soft hadron-hadron scattering cross-sections [10]. Below we shall consider the application of this language to a case of a massive colour projectile in a stochastic chromoelectric field. In this situation it is interesting to know, how the interaction with the medium changes the properties of the incident particle (for example, its energy, etc.). We shall consider the changes of the projectile energy due to the interaction with the stochastic chromoelectric external field. It will be shown that the particle can either lose or gain energy depending

on the model adopted for the description of the hadronic medium. The mechanism of this stochastic decceleration (acceleration) is completely analogous to the phenomenon of stochastic plasma heating well-known in plasma physics (see, e.g., [11]). In this case when the stochastic electric fields are present in the plasma (for example, this is a case for the turbulent plasma), the charged particles are accelerated by it. This model was succsefully applied to a description of the cosmic rays spectrum, where the charged particles are accelerated by a turbulent cosmic plasma (see, e.g, [12]).

2. Let us consider the propagation of a particle having a non-abelian charge in some representation of a gauge group in the external non-abelian field. The cooresponding quasi-classical equations of motion are well-known [13,14] and read

$$\left\{ \begin{array}{rcl} m\dot{u}_\mu & = & gQ^a(\tau)G^a_{\mu\nu}(x(\tau))\dot{x}^\nu \\ \dot{Q}_a(\tau) & = & -gf_{abc}\dot{x}^\mu(\tau)A^b_\mu(x(\tau))Q^c(\tau) \end{array} \right. \tag{1}$$

where the derivative in the left hand side is taken with respect to a proper time τ, m is a mass of an incident particle, g is a colour charge, Q_a are the components of a colour spin, f_{abc} are the structure constants of a corresponding Lie algebra, $x_\mu(\tau)$ is the particle trajectory in the space-time, $u^\mu(\tau)$ is a corresponding four-velocity, A^b_μ are the external gauge field potentials forming a stochastic ensemble and $G^a_{\mu\nu}$ is a corresponding field strength. The first equation describes the change of energy-momentum of the particle due to the interaction with the external field, and the second one describes the precession of the colour spin in the external nonabelian field.

In the following we shall confine our consideration to the one-dimensional case and look at the changes in the energy of the massive coloured projectile.

It is convenient to rewrite the system of equations (1) in the laboratory system:

$$\begin{array}{rcl} m\dfrac{dv_x}{dt} & = & gQ^a(t)E^a_x(t,x)(1-v_x^2)^{3/2} \\ \dfrac{d\mathcal{E}}{dt} & = & gQ^a(t)E^a_x(t,x)v_x \\ \dfrac{dQ^a}{dt} & = & gf_{abc}(v_xA^b_x(t,x)-A^b_0(t,x))Q^c(t), \end{array} \tag{2}$$

where the particle is moving along the x-axis with an instantoneous velocity $v_x(t)$, $E^a_x(t,x)$ is a chromoelectric field strength, $\mathcal{E}$ is energy of the particle. The calculations presented below are straightforwardly generalizing those by Sturrock [11] in his discussion of a stochastic plasma heating on a relativistic non-abelian case.

3. Let us now discuss the description of the ensemble of the stochastic chromoelectric field. We assume it is a stationary ensemble characterized by the lowest order correlators for

the group $SU(N_c)$:

$$\begin{cases} \langle E_x^a(t,x)\rangle & = & 0 \\ \langle E_x^a(t,x)E_x^b(t',x')\rangle & = & \frac{1}{3}\frac{1}{N_c^2-1}\langle \mathbf{E}^2\rangle \delta^{ab} R(t-t',x-x'), \end{cases} \tag{3}$$

where the brackets denote the averaging over the stochastic ensemble. The dimensionful quantity $\langle \mathbf{E}^2\rangle$ is an average value of the chromoelectric field squared (nonperturbative condensate). For the dimensionless correlation function R one has to introduce some parametrization. The Eucledian counterpart of this correlation was measured on the lattice [15]. Different parametrizations of it were used in phenomenological applications (I.V.Andreev [6], [8]).

Of a crucial importance for the following discussion is the *sign* of the condensate $\langle \mathbf{E}^2\rangle$. In the (by now) standard approach to the description of the nonperturbative phenomena in terms of condensates this quantity is *negative*. This shows itself, for example, in the unusual sign of the quadratic Stark effect due to the stochastic chromoelectric vacuum fields in heavy quarkonia [16]. At the same time the sources of chromoelectric fields like a system of heavy charges considered in the discussions of the QCD analog of the Landau-Pomeranchuk effect [17] or the instabilities of the quark-gluon plasma correspond to a *positive* sign of $\langle \mathbf{E}^2\rangle$. Below it will be shown, that it is precisely this difference in sign that leads to stochastic decceleration or acceleration of the projectile particle in the external stochastic chromoelectric field.

We conclude this section by some technical remarks. In the following we shall work in the lowest (second) order in the chromoelectric field and suppose that the autocorrelation time of the stochastic external field is less than a characteristic time scale of a problem (i.e. $\tau_c < 1$ fm). Another point is that as the third equation in (3) contains the gauge potentials it is necessary to express them through the electric field strength. The most straightforward way to do it is to use a Fock-Schwinger gauge $x^\mu A_\mu^a = 0$, in which the potentials are expressed through the field strength by the formula

$$A_\mu^a = \int_0^1 d\alpha \alpha x^\rho G_{\rho\mu}^a(\alpha x) \tag{4}$$

Then the equation for the charge evolution takes the form

$$m\frac{d}{dt}Q^a(t) = g f_{abc}(x+v_x t)\int_0^1 d\alpha\alpha E_x^b(\alpha t, \alpha x) Q^c(t) \tag{5}$$

which we shall use below.

4. Let us now look at the evolution of the average particle energy $\langle \mathcal{E}\rangle$. From the Eq. (2) we see that in order to calculate the right hand side of the second equation in the second order in $E^a(t,x)$ one has to calculate the changes in the coordinate, charge and velocity in

the first order in the chromoelectric field. Assuming the unperturbed motion of the form $x_0(t) = v_0 t$, one gets

$$\begin{aligned} x(t) &= v_0 t + \frac{g}{m} Q_0^a (1-v_0)^{3/2} \int_0^t dt'(t-t') E_x^a(t', v_0 t') \\ v_x(t) &= v_0 + \frac{g Q_0^a}{m} (1-v_0)^{3/2} \int_0^t dt' E_x^a(v_0 t', t') \\ Q^a(t) &= Q_0^a + \frac{2 g v_0}{m} f_{abc} \int_0^1 d\alpha \alpha \int_0^t dt' t' E_x^b(t', v_0 t') Q_0^c \end{aligned} \tag{6}$$

where Q_0^a is an initial colour spin. Substituting the formulas (6) into Eq.(2) and averaging over the stochastic external field, we get in the leading order in E^a:

$$\begin{aligned} \langle \frac{d\mathcal{E}}{dt} \rangle = \frac{4\pi\alpha_s \langle \mathbf{E}^2 \rangle}{3m(N_c^2-1)} \{ (1-v_0^2)^{3/2} (Q_0)^2 \int_0^\infty dt' R((t-t'), v_0(t-t')) \\ + (1-v_0^2)^{3/2} (Q_0)^2 \int_0^\infty dt' v_0 (t-t') R'((t-t'), v_0(t-t')) \\ + 2 v_0 f_{abc} \delta^{ab} Q_0^c \int_0^1 d\alpha \alpha \int_0^\infty dt' t' R(t-t', v_0(t-t')) \} \end{aligned} \tag{7}$$

where the derivative in the second term is taken with respect to a spatial coordinate and the infinite limit for the integration over time follows straightforwardly from the assumption that the autocorrelation time of the external field is small on the characteristic time scale of a problem.

From the antisymmetry of the structure constants f_{abc} it immediately follows that the third contribution is equal to zero and we finally get

$$\langle \frac{d\mathcal{E}}{dt} \rangle = \frac{4\pi\alpha_s \langle \mathbf{E}^2 \rangle}{3m(N_c^2-1)} (1-v_0^2)^{3/2} (Q_0)^2 \int_0^\infty d\tilde{t} (R(\tilde{t}, v_0\tilde{t}) + v_0 \tilde{t} R'(\tilde{t}, v_0 \tilde{t})) \tag{8}$$

The formula (8) clearly shows that the sign of the energy evolution is determined by that of an average square of the stochastic chromoelectric field (condensate) $\langle \mathbf{E}^2 \rangle$. For the usual nonperturbative condensate (negative $\langle \mathbf{E}^2 \rangle$) we have energy losses (stochastic decceleration), and in the opposite case of a positive $\langle \mathbf{E^2} \rangle$ we have the standard stochastic acceleration well-known in plasma physics.

In order to study the dependence of the energy loss (gain) of the colour projectile on the autocorrelation time of the stochastic chromoelectric field t_c we have to introduce a specific parametrization of the correlation function $R(t-t', x-x')$. Below we shall see that a choice of this parametrization can dramatically change the resulting expression for the energy losses.

Let us consider two possible parametrizations of the correlation function R:

$$R_1 = exp(-((t-t')^2 + (x-x')^2)/t_c^2) \tag{9}$$

$$R_2 = exp(-((t-t')^2 - (x-x')^2)/t_c^2) \tag{10}$$

Let us stress that the second Lorentz-invariant expression provides a natural analytical continuation of one of the parametrizations for the correlation function studied in Eucledian formalism. In Eq.(9) we have the correlation function R taken on the nonperturbed particle trajectory. Thus we get

$$R_1(\tilde{t}) = exp(-(1+v_0^2)\tilde{t}^2/t_c^2) \tag{11}$$

$$R_2(\tilde{t}) = exp(-(1-v_0^2)\tilde{t}^2/t_c^2) \tag{12}$$

Substituting these parametrization into Eq. (9) we get

$$\langle\frac{d\mathcal{E}}{dt}\rangle_1 = \frac{2\pi^{3/2}\alpha_s\langle E^2\rangle\tau_c}{3m}(\frac{1-v_0^2}{1+v_0^2})^{3/2}\frac{Q_0^2}{N_c^2-1} \tag{13}$$

and

$$\langle\frac{d\mathcal{E}}{dt}\rangle_2 = \frac{2\pi^{3/2}\alpha_s\langle \mathbf{E}^2\rangle\tau_c}{3m}\frac{Q_0^2}{N_c^2-1} \tag{14}$$

We can conclude that the usual description of the hadronic medium (negative $\langle\mathbf{E}^2\rangle$) leads to the energy loss of a colour projectile in the stochastic colour medium thus *softening* the energy spectrum of the produced hadrons. At the same time the stochastic acceleration of heavy quarkonium by stochastic fields in quark-gluion plasma originating from its instabilities or even turbulence will *harden* this spectrum.

Denoting the initial energy of the projectile by E_0 and recalling, that the conserved square of the colour spin is equal to [14]

$$(Q_0^a)^2 = 3C_A$$

where C_A is a Casimir operator of the corresponding representation of the colour qroup (for $SU(N_c)$ $C_V = N_c$, $C_F = (N_c^2-1)/2N_c$ for the vector and fundamental representations respectively) we obtain the final formulas

$$\langle\frac{d\mathcal{E}}{dt}\rangle_1 = \frac{2\pi^{3/2}\alpha_s\langle \mathbf{E}^2\rangle\tau_c}{m}(\frac{2E_0^2}{m^2}-1)^{-3/2}\frac{C_A}{N_c^2-1} \tag{15}$$

and

$$\langle\frac{d\mathcal{E}}{dt}\rangle_2 = \frac{2\pi^{3/2}\alpha_s\langle \mathbf{E}^2\rangle\tau_c}{m}\frac{C_A}{N_c^2-1} \tag{16}$$

We see that the energy dependence of the energy losses rate has a dramatic dependence on the form of the correlation function.

For the intuitively appealing (although Lorentz-noninvariant) choice R_1 the rate of the energy losses (gains) decays (at sufficiently high energies of the incident particle E_0) as E_0^{-3} and thus is numerically negligible.

The second Lorentz-invariant choice R_2 leads to an energy independent energy loss (gain) rate and thus can be quantitatively important.

Let us estimate the corresponding energy losses for the charmed quark (fundamental representation of $SU(3)$) and the J/Ψ meson in the octet state (adjoint representation of $SU(3)$) in the case of a constant energy change rate. Using

$$\langle \alpha_s \mathbf{E}^2 \rangle = -\frac{1}{4} \langle \alpha_s G^2 \rangle,$$

$m_c = 1.5$ GeV , $\langle \alpha_s G^2 \rangle = 0.042$ GeV^4 and $t_c = 0.3$ fm we get

$$\langle \frac{d\mathcal{E}}{dt} \rangle_q = 98 \frac{\mathrm{Me}V}{\mathrm{f}m}$$

and

$$\langle \frac{d\mathcal{E}}{dt} \rangle_{J/\Psi} = 110 \frac{\mathrm{Me}V}{\mathrm{f}m}$$

Taking into account the additional numreical uncertainities (the value of t_c, etc) we see, that the nonperturbative decceleration of the described type can be around 10-20 percent with respect to a naively expected stringy energy loss in the hadronic medium.

Concluding this section we note, that the formulas (15) and (16) illustrate (within logarithmic accuracy) the borderline energy dependences following from the simplest choice of a correlation function characterizing the ensemble of the stochastic chromoelectric external fields.

5. Let us briefly summarize the results. Considering the quasiclassical equations of motion for a heavy coloured particle propagating in the external stochastic chromoelectric field we have calculated the energy evolution rate. The calculations are generalizing to a non-abelian relativistic case those explaining the stochastic heating phenomenon in plasma physics. We have shown that the cases of a projectile decceleration and acceleration are distinguished by the sign of an chromoelectric field condensate. The negative sign corresponds to stochastic decceleration (energy loss) and the positive one to stochastic acceleration (energy gain). The energy dependence of the rate of the energy loss turns out to be very sencitive to the form of the correlation function of the stochastic external chromoelectric field.

The first possibility (negative $\langle E^2 \rangle$) is believed to be realized in the nonperturbative colour medium leading for example to an unusual sign of a level shift of heavy quarkonium due to a quadratic Stark effect [16]. The experimental prediction will be the corresponding softening of the energy spectrum of the produced hadrons.

The most natural situation when the second possibility (positive $\langle E^2 \rangle$) is realized is for hadron production in the quark-gluon plasma in the presence of an instability (turbulence)

or a propagation of coloured projectile originating from the hard stage of a collision through it. The experimental prediction is the hardening of the energy spectrum of the produced hadrons. This phenomenon is analogeous to the stochastic acceleration of charged particles in the cosmic plasma responcible for the formation of the energy spectrum of cosmic rays [12].

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